The analysis of temperature fluctuations by pulse-counting techniques

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To determine experimentally the mean value of a randomly fluctuating quantity, it may be necessary to measure the average value over a considerable interval of time. This problem arose in a recent study of the temperature fluctuations over a heated horizontal plate, and a system was used that depended on the counting of electrical pulses generated at a rate proportional to the quantity being measured. The advantage of this technique is that mean values may be measured over time intervals of almost unlimited length with little added difficulty for the experimenter. Circuits are described which measure: (*a*) the mean square of a fluctuating quantity and of its time-derivative, (*b*) the statistical distribution of the fluctuations, (*c*) the mean frequency of the fluctuation assuming a particular value, and (*d*) the mean product of two fluctuating quantities. Over the range of use, the stability and linearity of the calibrations is better than 1 %, more than sufficient for work on natural convection. In its present form, the equipment responds uniformly to all frequencies below 100 c/s, but it would not be difficult to extend this range of response to higher frequencies.

1. Introduction

It has been recognized since the time of Reynolds that the fluctuations occurring in turbulent flow are so irregular that no significance can be attached to individual measurements and that theoretical and experimental investigations must be made in terms of mean values (or statistical expectations) of a small group of functions of the fluctuations. The particular functions chosen do not form a complete statistical specification as their choice is dictated by reasons of theoretical or experimental convenience, but it is usual to hope that they specify the most significant aspects of the flow. Most laboratory work concerns turbulent flows that are statistically stationary in time and then the mean values are obtainable, in principle, as time-averages over periods sufficiently long compared with the time-scale of the fluctuations. In practice, the effective duration of the averaging process depends on the technique of measurement and one adequate for measuring the rapid fluctuations in a wind-tunnel (e.g. use of a continuously reading instrument with a response-time of about 5 sec) may be nearly useless for the much slower fluctuations occurring in the atmosphere or in systems of natural convection. With these and similar flows, it may be necessary to average over periods of 10 min or more, which requires that either the measuring equipment or the experimenter must 'remember' all the fluctuations over this time. An obvious technique is to record the fluctuations on magnetic tape or on photographic paper and to analyse them later, but recording is an unnecessary complication of laboratory work unless there is a special advantage in having a time record, e.g. the possibility of determining the auto-correlation function. A more convenient method is to generate electrical pulses at a rate proportional to the quantity whose mean value is required and, by counting the total number of pulses emitted during a suitably long interval of time, to obtain an approximation to the mean value.

This paper describes a group of pulse-counting techniques that were developed for a recent study of the temperature fluctuations over a heated horizontal plane (Townsend 1959). With the appropriate combinations of the electrical circuits, it was possible to measure mean values, mean squares and statistical distributions of either the temperature, the temperature gradient or the time rate-of-change of temperature. The actual analysing equipment is loosely based on a system built and used by Thomas (1956) for measuring the auto-correlation function of the temperature fluctuations by observing the rate of coincidences between two frequency-modulated trains of pulses, and the availability of this equipment and knowledge of its qualities and defects were of considerable assistance in the development of the present system. Similar methods have been used in studies of turbulence, by Corrsin & Kistler (1954) and by others, but, to my knowledge, they have not been used to measure the group of quantities that are commonly measured with the hot-wire anemometer and its associated equipment.

2. The resistance thermometer

The fluctuations of temperature are detected as changes in resistance of a short length of platinum wire, 1–2 mm long and $2{\cdot}5\,\mu$ in diameter, which forms one arm of a Wheatstone bridge. The bridge is supplied with alternating current

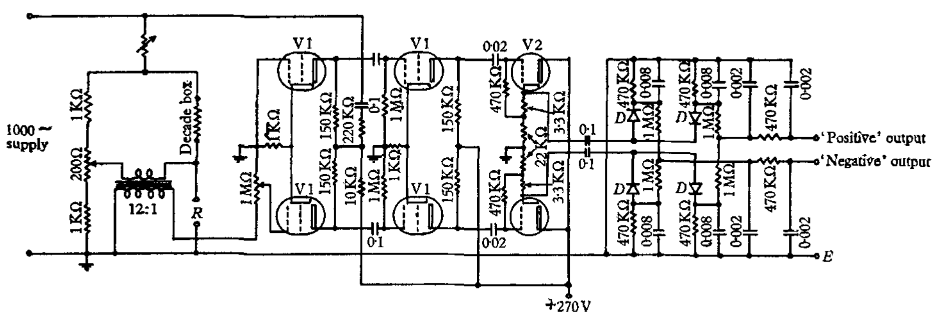

FIGURE 1. Circuit diagram of Wheatstone bridge and amplifier for use with resistance thermometer. (V1 type EF37, V2 type ECC35; D, crystal rectifier; R, thermometer element.)

at 1000 c/s and the output is amplified and converted by a phase-sensitive rectifier to an output directly proportional to the resistance unbalance of the bridge (figure 1). The output is balanced with respect to earth, that is, there are two output leads whose potentials differ by an amount proportional to the bridge unbalance but have a sum equal to ground potential. They will be distinguished by calling them the 'positive' output and the 'negative' output, the 'positive' output being the one which has a positive potential when the resistance of the thermometer element exceeds the balance resistance. After passing through the necessary filter circuits to reduce the alternating-current ripple to a negligible amount, the electrical output responds to resistance changes with a lag of about 2 msec. This is sufficiently rapid for measurements of natural convection, although it would not be difficult to improve the speed of response by using alternating current of higher frequency.

3. Measurement of mean values

The production of pulses at a rate proportional to the resistance unbalance of the Wheatstone bridge is conveniently achieved by a modification of a familiar time-base circuit (figure 2). The condenser C is charged by the anode current of a pentode which is connected so that this current is very nearly proportional to

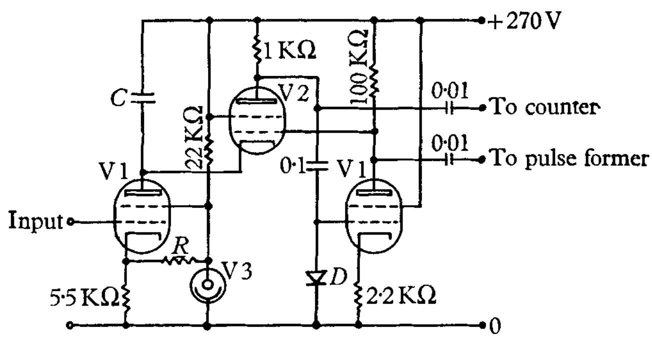

FIGURE 2. Circuit diagram of primary pulse generator. (V 1 type CV 138, V 2 type CV 416; D, crystal rectifier.) The adjustment of C and R is described in the text. Approximately, $C = 0{\cdot}05\,\mu$F, $R = 250\,$KΩ.

the potential of the control grid if this potential is positive and zero if it is negative. When the charge on the condenser reaches a critical value, the trigger circuit is excited and discharges the condenser. If the discharge process is instantaneous, the trigger circuit emits an electrical pulse whenever the total charge transmitted by the charging pentode has increased by a definite amount, that is, whenever the time-integral of the *positive* values of the input has increased by a fixed amount. The total number of pulses emitted is so proportional to the time-integral of the positive values of the input over the time of observation.* If one such circuit is used with each side of the output of the thermometer system (the 'positive' and 'negative' outputs mentioned in the previous section), the difference between the total numbers of 'positive' and 'negative' pulses is proportional to the time integral of the output and so to the average out-of-balance of the Wheatstone bridge over the time of counting. Obviously this requires nearly identical performance of the two pulse circuits, and this is secured

* In use, the total number of pulses recorded is always so large that the unregarded 'fractions' at the beginning and end of the counting periods are a negligible part of the total.

by adjustment of the resistances *R* and the condensers *C* (figure 2). The resistances are adjusted until the pulse rate is proportional to input and then one condenser is adjusted to make the constants of proportionality the same in the two circuits.

4. Measurement of mean squares and mean products

The measurement of the mean square of a fluctuating input depends on constructing from each pulse emitted by the primary pulse circuits a pulse of square wave-form and standard duration (approximately 1 msec). These pulses are

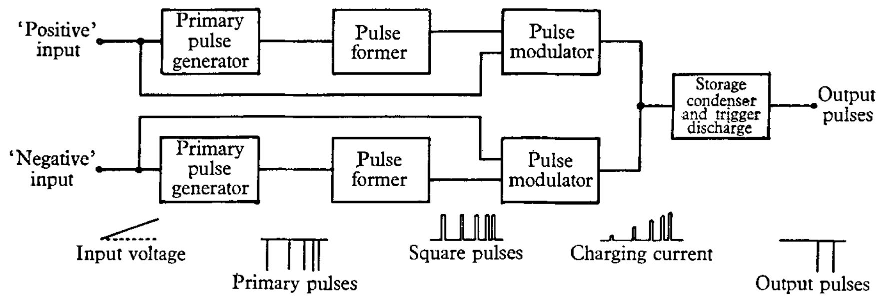

FIGURE 3. Block diagram of circuit for measuring mean squares, showing wave-forms in the 'positive' channel for the indicated 'positive' input.

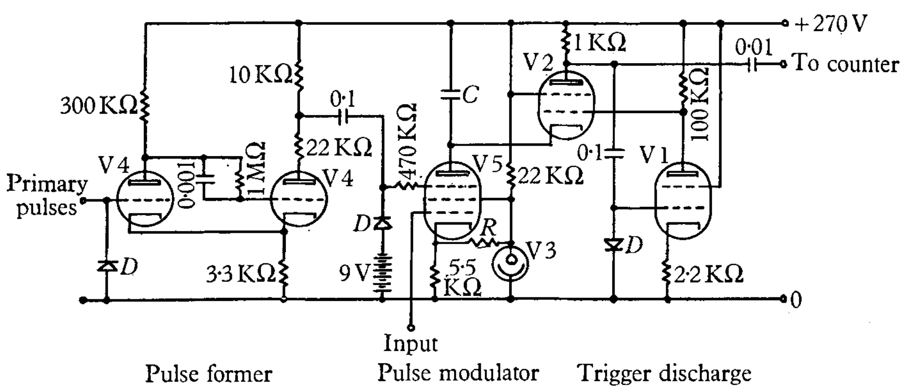

FIGURE 4. Details of circuit for measuring mean squares. (V1 type CV138, V2 type CV416, V3 type CV287, V4 type CV858, V5 type CV2209.) N.B. The condenser *C* is also charged by modulated pulses transmitted by a 'negative' channel not shown here.

used to switch on for their duration a current proportional to the input that produced them, and these modulated pulses of current charge a condenser which, like the condenser in the primary pulse circuit, is discharged when its potential reaches a critical value. The distinction between 'positive' and 'negative' inputs and the corresponding pulses makes necessary a duplication of the pulse-forming and pulse-modulation circuits but the modulated pulses of current from both channels are used to charge the same condenser, as is shown in the block diagram (figure 3). Details of the circuits are shown in figure 4. A simple way of looking at the working of this arrangement is to say that the storage condenser is receiving pulses of current whose recurrence frequency and intensity are both proportional to the input and to infer that the number of discharges in a time interval is proportional to the average square of the input. This view makes it appear that the equipment will not measure accurately the mean square of an input that varies appreciably in a time comparable with the interval between successive primary pulses. That this is not correct may be seen from the following argument. The probability that (say) the 'positive' channel should produce a primary pulse in a short time-interval is proportional to the input at that time, and a primary pulse, when it occurs, leads to the storage condenser receiving a charge proportional to the time integral of the input over a subsequent time, τ, equal to the duration of the 'square' pulse. Consequently, the expected charge received by the condenser in unit time is proportional to

$$\overline{e(t)\int_t^{t+\tau} e(t')\,dt'} = \int_0^\tau R(\theta)\,d\theta, \tag{4.1}$$

where $R(\theta) = \overline{e(t)\,e(t+\theta)}$ is the covariance between values of the input, separated in time by an interval θ. If τ is small compared with the time-scale of the fluctuations, the expected charge per unit time and so the expected pulse-rate are proportional to $\overline{e^2(t)}$, the mean square of the input. This condition is usually satisfied.

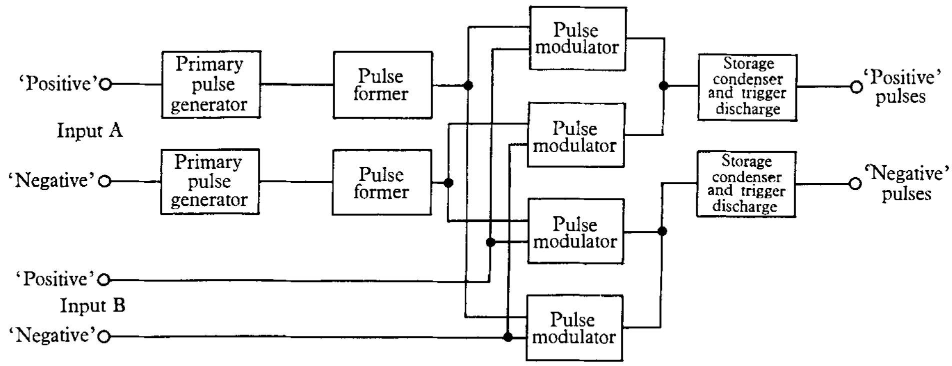

FIGURE 5. Block diagram of circuit for measuring mean products.

It is necessary that the performances of the two channels be the same if the system is to work in the manner described. This requires that the durations of the pulses from each pulse-forming circuit should be the same and that the heights of the modulated pulses should be the same for the same inputs. The first requirement is satisfied by adjustment of the coupling condenser in the pulse-forming circuit, and the second by using a common cathode resistance for the two pulse modulators and by approximate matching of the valves.

This method of measuring the mean square of a fluctuating signal is easily adapted for the measurement of the mean product of two fluctuating signals. For this, it is necessary to use two storage condensers, one receiving current pulses at a mean rate proportional to the mean of the positive values of the product and the other at a mean rate proportional to the mean of the negative values. The whole arrangement is shown as a block diagram in figure 5.

5. Performance and errors

The actual performance of these circuits departs from the ideal in several respects. In the first place, the variation of charging current of the storage condenser with input voltage is not linear for inputs less than 2 V and this leads to counting rates slightly above the 'ideal' values. Secondly, the trigger circuits do not work if the charging currents are too small, and the emission of pulses ceases completely if the input is less than 0·5 V. Thirdly, the circuit is insensitive and ignores input fluctuations during periods of condenser discharge. The first two defects concern the response at low inputs and low counting rates, and their consequences are negligible if the input fluctuations cover a sufficient range. The insensitivity during the time of condenser discharge reduces high counting rates in the ratio $(1+nt_d)^{-1}$ (where n is the 'ideal' counting rate and t_d is the discharge time). Since t_d is roughly 0·05 msec, this is a negligible reduction, even at the counting rate for the maximum input (nearly 300 sec^{-1}). These defects are also present in the operation of the squaring and multiplying circuits but have even less effect on the measurements obtained.

The ability of the squaring and multiplying circuits to respond to high-frequency components (that is, to include their contribution in the counting rate) depends mostly on the duration of the 'square' pulses, as is shown by equation (4.1). For a pulse duration of 1 msec, the response at 250 c/s is 65 % of the low-frequency response, and is 93 % at 100 c/s.

Other errors may arise from instability of the necessary adjustments and changes in the sensitivity of the various elements of the system. In the design, efforts have been made to relate the performance of the various circuits to values of resistances rather than to valve characteristics, and the long-term stability is surprisingly good with overall calibrations repeating from one day to another within $\frac{1}{2}$ %. It perhaps should be pointed out here that the influence of these variations on the actual measurements is reduced very considerably by the use of 'positive' and 'negative' channels. If only one channel were used, it would be necessary to restrict the input to positive values and then the mean square of the fluctuation from the mean would be obtained as the difference of two counting rates of similar magnitude. For example, if the fluctuations are normally distributed, the standard deviation may not exceed 0·4 of the mean if negative excursions are to be sufficiently improbable and the ratio of the mean square to the square of the mean will lie between 1 and 1·16. As the mean-square deviation from the mean is computed from the difference between an observed counting-rate proportional to the mean square and a calculated counting-rate proportional to the square of the mean, the instrumental uncertainty in this difference will be about ten times greater than the uncertainty in the basic calibrations. Using a second channel for negative excursions of the input allows adjustment of the input so that the square of the mean is small compared with the mean square, and then the uncertainty in the measurement of the mean-square fluctuation is of the same order of magnitude as the uncertainty of the calibration.

In use, the pulses emitted by the two primary pulse circuits and by the 'squaring' circuit are counted simultaneously, and the total numbers recorded in a measured time-interval are used to calculate the average and the average square of the fluctuation from an arbitrary but known zero, taken over the same time-interval. The probable deviations of these averages from the true mean

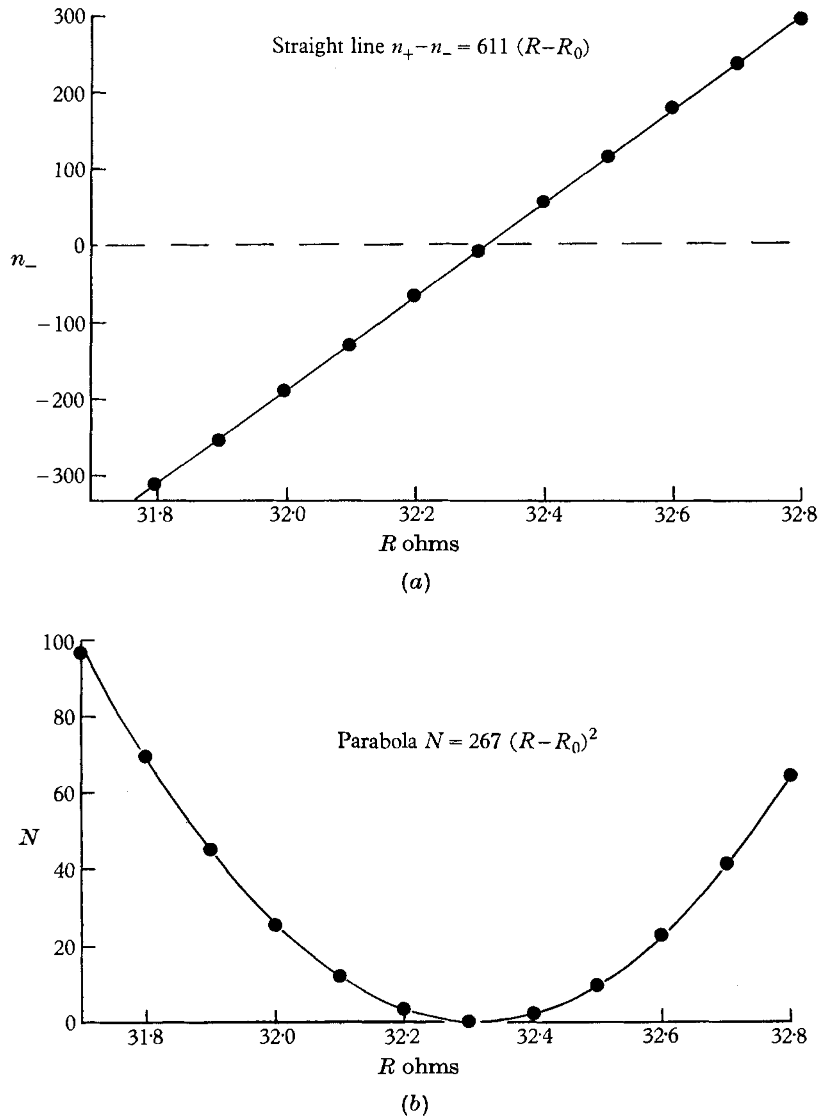

FIGURE 6. Typical calibration curves. (n_+, n_- are the pulse rates in the 'positive' and 'negative' channels, N is the pulse rate from the 'squaring' circuit, and R is the resistance in ohms of the thermometer element.)

values depend on the ratios of the time-scales of the fluctuations to the period of counting, but this statistical uncertainty is exactly what would arise if a time-lapse record of the same duration were analysed. In effect, the electrical system carries out the numerical analysis as the information arrives, rather than later.

6. Measurement of the distribution of fluctuations

The mean square of the fluctuation from the mean is a convenient but incomplete measure of the fluctuations, and useful information is often obtainable from the statistical distribution of the fluctuation amplitudes. This may be measured with the circuit shown in figure 7. In this, a square-wave of large amplitude and recurrence frequency 50 c/s is applied to the suppressor grids of five similar pentodes and switches the anode currents on and off at the same frequency. The magnitude of each anode current is determined by the potential

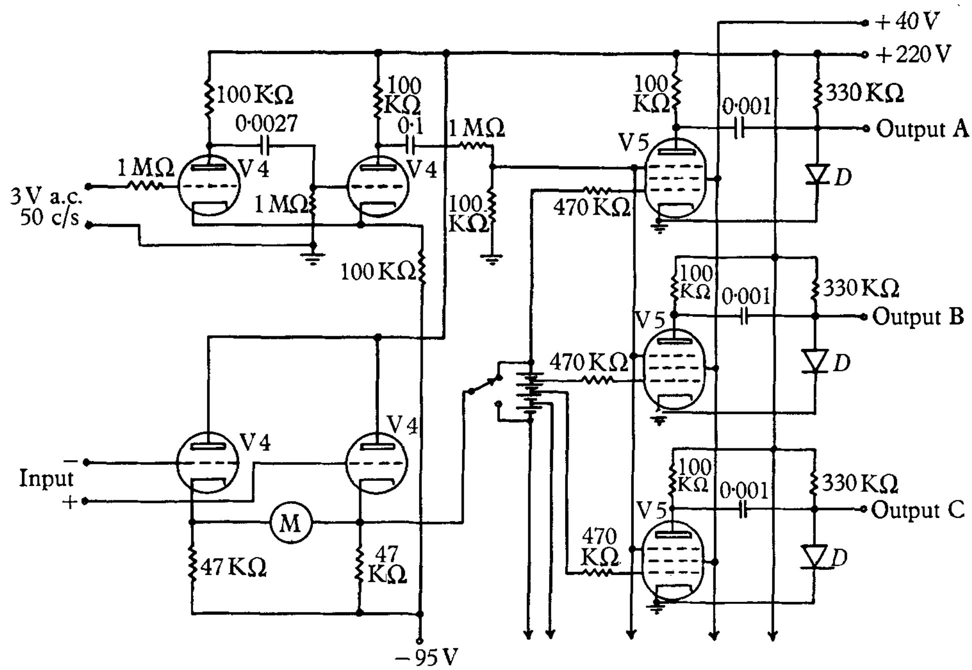

FIGURE 7. Details of circuit for the measurement of statistical distributions of input fluctuations. (V 4 type CV 858, V 5 type CV 2209; *D*, crystal rectifier. N.B. Only three of the five pentodes are shown. The voltmeter *M* gives a visual indication of the slower fluctuations.)

of the control grid, and only if this potential exceeds a critical value is the variation of anode current (caused by the switching) sufficient to transmit through the diode discriminator pulses of sufficient strength to work a dekatron counter. By means of a cathode follower, each grid potential is linearly related to the input and each pentode begins to transmit countable pulses when the input potential exceeds a critical value. By the use of bias cells, the five critical values are separated by intervals of approximately 1·5 V, and a switching system (only partially shown in figure 7) allows the five critical potentials to be changed together so that a range of critical potentials from − 10·5 V to + 10·5 V is available. Pulses are transmitted by each pentode during the time that the input potential exceeds the critical value, and the total number counted is proportional to the fraction of the time of observation that the input exceeded the critical value, which is an approximation to the probability that a fluctuation should exceed this value. Five such probabilities can be measured simultaneously and twenty can be measured in four successive runs without alteration of the thermometer bridge.

Naturally, the transmission of countable pulses does not begin suddenly at the critical potential and there is a transition range, due in part to residual ripple at 1000 c/s. The change from zero to maximum counting-rate occurs over a range of input potential that is about 10 % of the spacing between successive critical potentials (figure 8), but this resolution is ample for the analysis of ordinary distributions.

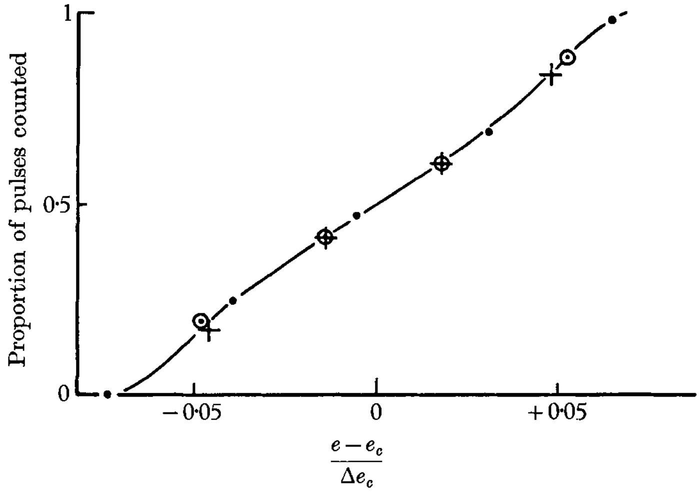

FIGURE 8. Resolution curve for the statistical analyser. (e_c is the potential at which half the pulses are counted, and Δe_c is the difference between successive values of e_c.)

7. Measurement of the time-derivative of a fluctuation

It is of interest to study the rate of change of fluctuations and this may be done in two ways. The first and more comprehensive method is to construct from the thermometer output an electric signal proportional to the time-rate-of-change of the output and to analyse this signal with the circuits described above. This has been done with a standard circuit (Townsend 1947) modified to work accurately at very low frequencies (figure 9). The relation between input and output is

$$\text{Output} = \tau_0 \frac{d}{dt}(\text{input}), \tag{7.1}$$

where

$$\tau_0 = \frac{\alpha\mu}{\mu+1} C_2 \frac{R_2 R_3}{R_1},$$

and μ is the amplification factor of the pentodes, connected as triodes,

α is the fraction of the total cathode current reaching the anode,

and the circuit components have been selected so that

$$2C_1 R_1 = C_2(R_2+R_3). \tag{7.2}$$

In the actual circuit, the characteristic time, τ_0, was calculated to be 0·0309 sec, using the measured valve characteristics and component values.

The second method is very simple to use and depends on measuring the frequency with which the fluctuation from the mean changes sign from negative to positive. This frequency is

$$N_0 = \frac{1}{2\pi}\left[\overline{\left(\frac{de}{dt}\right)^2}\Big/\overline{e^2}\right]^{\frac{1}{2}} \tag{7.3}$$

if the fluctuations and their rates-of-change are statistically independent and normally distributed (Rice 1945). A simple circuit that produces a pulse whenever

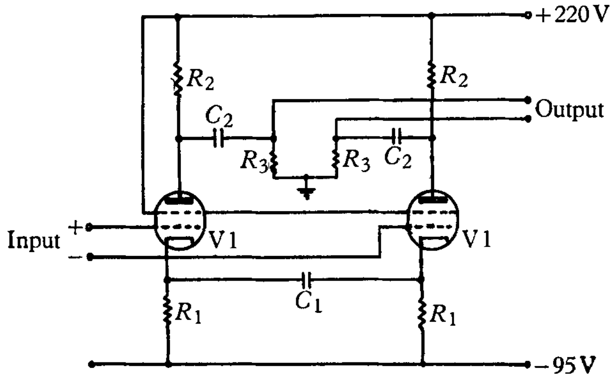

FIGURE 9. Circuit for differentiation with respect to time. (V 1 type CV 138, actual values $R_1 = 9{\cdot}8 \times 10^4\,\Omega$, $R_2 = 9{\cdot}9 \times 10^4\,\Omega$, $R_3 = 1{\cdot}00 \times 10^6\,\Omega$, $C_1 = 0{\cdot}220\,\mu$F, $C_2 = 0{\cdot}0396\,\mu$F.)

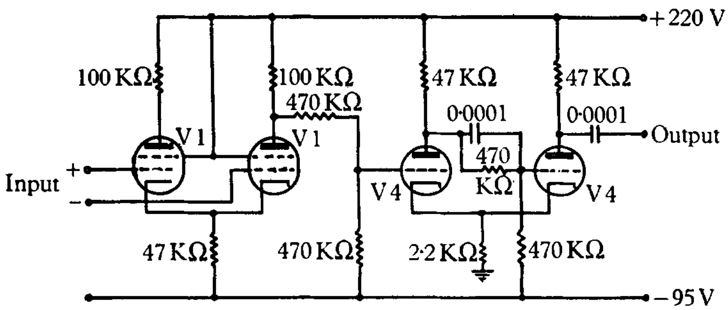

FIGURE 10. Circuit for counting zero crossings. (V 1 type CV 138, V 4 type CV 858.)

the input passes through a critical value is shown in figure 10. It consists of a directly-coupled amplifier whose output controls a Schmitt trigger circuit whose current distribution changes discontinuously at a critical grid voltage (Farley 1955, p. 58). Neglecting a small amount of 'backlash', the right-hand triode conducts whenever the input is less than the critical value and is non-conducting when it exceeds this value. Consequently, the small condenser connected to its anode transmits a positive pulse as the input increases through the critical value and a negative pulse as it decreases through the value. The purpose of the amplifier is to reduce the effective backlash to a negligible amount. Owing to the difficulty of interpreting the results if the statistical distributions are not normal, this method is less useful than the first for the study of fluctuations of unknown characteristics, but its simplicity makes it a valuable addition to the whole system of measurement.

8. Calibration and pulse-counting

The pulses originating in the various circuits are counted with a five-channel, scale-of-1000, electrical counter, using three stages of dekatrons and a mechanical counter. The various calibrations were made by substituting for the resistance-thermometer element a decade resistance box, covering the range, 10–110 Ω, in steps of 0·01 Ω, and observing the counting rates.

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